Approximation of the biharmonic problem using P1 finite element

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Key Words: *P1 finite element approximation; Biharmonic problem; Numerical results.*

Abstract:
We study in this paper a P1 finite element approximation of the solution in $H_0^2(\Omega)$ of a biharmonic problem. Since the P1 finite element leads only to an approximate solution in $H_0^1(\Omega)$, a discrete laplacian operator is used in the numerical scheme. The convergence of the method is shown, for the general case of a solution whose regularity is not greater than $H_0^2(\Omega)$, thanks to compactness results and to the use of a particular interpolation of regular functions with compact supports. An error estimate is proved in the case where the solution is in $C^4(\overline{\Omega})$. The order of this error estimate is equal to 1 if the solution has a compact support, and only 1/5 otherwise. Numerical results show that these orders are not sharp in particular situations.

Références

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